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$$\sum_{n=1}^{\infty}\frac{U_n}{r^n}=\frac{1}{\sqrt{5}}\left(\frac{a+b\rho_1}{\rho_1(r-\rho_1)}-\frac{1}{\sqrt{5}}\frac{a+b\rho_2}{\rho_2(r-\rho_2)}\right).$$

The right-hand member now reduces to

$$\frac{b+ar-a}{r^2-r-1}$$

when ρ_1 is replaced by $(1+\sqrt{5})/2$ and ρ_2 by $(1-\sqrt{5})/2$.

Note.—We are publishing this solution for the reason that the previously published solution referred to did not consider the question of convergency correctly, and the proper investigation of this question was the Proposer's chief reason for proposing the problem. EDITORS.

ALGEBRA.

429. Proposed by C. N. SCHMALL, New York City.

It is given that d_1, d_2, d_3 are the greatest common divisors of y and z, z and x, x and y, respectively; also that m_1, m_2, m_3 are the least common multiples of the same pairs of members. If d and m are the greatest common divisor and least common multiple, respectively, of x, y, and z, show that

$$\frac{m}{d}=\left(\frac{m_1m_2m_3}{d_1d_2d_3}\right)^{\frac{1}{2}}.$$

SOLUTION BY FRANK IRWIN, University of California.

It is evident that we can get the least common multiple of two numbers by dividing their product by their greatest common divisor:

$$m_1=\frac{yz}{d_1},\quad m_2=\frac{xz}{d_2},\quad m_3=\frac{xy}{d_3}.$$

Similarly with the three numbers x, y, z, if we divide their product by $d_1d_2d_3$, we should have their least common multiple, except that we have divided out d once too often:

$$m=\frac{xyz}{d_1d_2d_3}\cdot d.$$

We have then:

$$\left(\frac{m_1}{d_1},\frac{m_2}{d_2}\cdot\frac{m_3}{d_3}\right)^{\frac{1}{2}}=\left(\frac{yz}{d_1^2}\cdot\frac{zx}{d_2^2}\cdot\frac{xy}{d_3^2}\right)^{\frac{1}{2}}=\frac{xyz}{d_1d_2d_3}=\frac{m}{d}.$$

Also solved by A. H. HOLMES, ELMER SCHUYLER, G. W. HARTWELL, FRANK R. MORRIS, N. P. PANDYA, HERBERT N. CARLETON, PAUL CAPRON, J. A. CAPARO, and the PROPOSER.

GEOMETRY.

443. Proposed by C. N. SCHMALL, New York City.

A quadrilateral of any shape whatever is divided by a transversal into two quadrilaterals. The diagonals of the original figure and those of the two resulting (smaller) figures are then drawn. Show that their three points of intersection are collinear.

III. SOLUTION BY LAENAS G. WELD, Pullman, Ills.

To the triangle ABC draw the transversals MN, intersecting AB in M and AC in N, and QR intersecting AB in Q and AC in R. Then $BCRQ$, $BCNM$ and

QRNM are the three quadrilaterals in question. Designate the intersection of the diagonals BN and CM by F, that of BR and CQ by G, and that of QN and RM by H. Then F, G, and H are collinear?

In vectors, let $AB = \beta$, $AC = \gamma$; $AM = m\beta$, $AN = n\gamma$; $AQ = q\beta$, $AR = r\gamma$. Then

$$MC = -m\beta + \gamma \quad \text{and} \quad MF = x(-m\beta + \gamma),$$

in which x is to be determined. Also

$$NB = -n\gamma + \beta \quad \text{and} \quad NF = y(-n\gamma + \beta),$$

in which y is to be determined. Now

$$m\beta + x(-\beta m + \gamma) = AF = n\gamma + y(-n\gamma + \beta); \tag{1}$$

or

$$m(1 - x)\beta + x\gamma = n(1 - y)\gamma + y\beta,$$

in which the coefficients of β and γ may be equated. Thus

$$mx + y = m,$$

$$x + ny = n.$$

Solving:

$$x = \frac{n(m-1)}{mn-1}, \quad y = \frac{m(n-1)}{mn-1};$$

whence, by Eq. (1),

$$AF = \frac{m(n-1)}{mn-1} \cdot \beta + \frac{n(m-1)}{mn-1} \cdot \gamma. \tag{2}$$

Similarly:

$$AG = \frac{q(r-1)}{qr-1} \cdot \beta + \frac{r(q-1)}{qr-1} \cdot \gamma, \tag{3}$$

and

$$AH = \frac{mq(n-r)}{mn-qr} \cdot \beta + \frac{nr(m-q)}{mn-qr} \cdot \gamma. \tag{4}$$

By the theory of vectors, if there are three multipliers, λ, μ, ν, such that

$$\lambda \cdot AF + \mu \cdot AG + \nu \cdot AH = 0,$$

while

$$\lambda + \mu + \nu = 0,$$

the vectors AF, AG, AH terminate collinearly. This requires, by Equations (2), (3), (4), the consistency of the equations

$$\frac{m(n-1)}{mn-1} \cdot \lambda\beta + \frac{q(r-1)}{qr-1} \cdot \mu\beta + \frac{mq(n-r)}{mn-qr} \cdot \nu\beta = 0,$$

$$\frac{n(m-1)}{mn-1} \cdot \lambda\gamma + \frac{r(q-1)}{qr-1} \cdot \mu\gamma + \frac{nr(m-q)}{mn-qr} \cdot \nu\gamma = 0,$$

$$\lambda \quad + \quad \mu \quad + \quad \nu = 0.$$

This consistency is established by the vanishing of the determinant

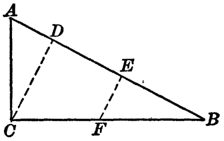
$$\Delta \equiv \begin{vmatrix} m(n-1) & q(r-1) & mq(n-r) \\ n(m-1) & r(q-1) & nr(m-q) \\ mn-1 & qr-1 & mn-qr \end{vmatrix};$$

which proves the proposition.

Note.—Solutions of this problem appeared in the January and May numbers of the MONTHLY, presenting two different methods of attack. The one here given presents still a different method.
EDITORS.

459. Proposed by C. N. SCHMALL, New York City.

In a right triangle *ABC*, right-angled at *C*, a point *F* is taken in the side *CB*, and perpendiculars *CD* and *FE* are dropped on the hypothenuse *AB*.



Prove that

$$AD \cdot AE + CD \cdot EF = \overline{AC}^2.$$

SOLUTION BY HERBERT N. CARLETON, West Newbury, Mass.

Triangles *ACD, CDB, EFB,* and *ACB* are similar. Hence,

$$AD : CD = EF : EB;$$

whence

$$AD \cdot EB = CD \cdot EF.$$

Similarly

$$AD : AC = AC : AB;$$

whence

$$\overline{AC}^2 = AB \cdot AD = (AE + EB)AD = AD \cdot AE + AD \cdot EB.$$

Substituting for $AD \cdot EB$, we have

$$\overline{AC}^2 = AD \cdot AE + CD \cdot EF.$$

Also solved by A. M. HARDING, J. W. CLAWSON, ELMER SCHUYLER, FRANK IRWIN, WALTER C. EELLS, L. G. WELD, J. A. CAPARO, A. H. HOLMES, NATHAN ALTSHILLER, G. W. HARTWELL, HORACE OLSON, and HARMON ANNING.

460. Proposed by J. W. CLAWSON, Ursinus College, Pa.

ABC is a triangle, *O* and *I* the centers of the circum- and in-circles respectively, and I', I'', I''' the centers of the three escribed circles. If *AO, BO, CO* meet the circumcircle in P, P', P'' respectively, and PR, $P'R'$, $P''R''$ are drawn parallel respectively to *AI, BI, CI* to meet *BC, CA, AB* respectively in R, R', R'', prove that: (1) PR, $P'R'$, $P''R''$ are concurrent, say at *J*. (2) $JO = OI$. (3) JI', JI'', JI''' are perpendicular respectively to *BC, CA, AB*. (4) AR, BR', CR'' are concurrent.

I. SOLUTION BY THE PROPOSER.

(1) Extend *IO* its own length to *J*. Join *JP*. Then △'s *AOI, POJ* are congruent. Hence, $\angle APJ = \angle PAI$. Hence *J* lies on *PR*. Similarly, *J* can be shown to lie on $P'R'$ and on $P''R''$.